

# pBUU Description

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National Superconducting Cyclotron Laboratory  
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on Transport Simulations for Heavy Ion Collisions  
under Controlled Conditions

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# pBUU Features

- **Solution of Boltzmann Eq**
- 1-Ptcle Energies from Energy Functional
  - Volume (incl Momentum), Gradient, Isospin, Coulomb Terms
- Covariance
  - Covariant: Volume (incl Momentum) Term in Energy, Collisions
  - Noncovariant: Gradient, Isospin, Coulomb Terms in Energy
  - Employed (so far) up to 20 GeV/nucl
- Pions contribute to Symmetry Energy
- Spectral functions of  $\Delta$  and  $N^*$  Resonances in adiabatic approximation
  - Detailed Balance for Broad Resonances
- $A = 2, 3$  Clusters produced in Multinucleon Collisions
  - Cluster Break-Up Data used in describing Production



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## pBUU - Technical Aspects

- Initial State from Solving Thomas-Fermi Eqs
- Wigner Functions represented in term of Test Particles
- Lattice Hamiltonian (Lenk & Pandharipande)
  - Profile Functions associated with Lattice Nodes
  - Test-Particle Eqs of Motion from the Lattice Hamiltonian
  - Values of Hamiltonian and Net Momentum Conserved
- Collisions, including Multiparticle, between *Any* Test-Particles within Spatial Cell
- Computational Speed Enhanced processing only Collision No that may be occur within Time-Step
- Occupations  $f$ /Pauli Principle: (a) smoothing Test-Particles, *in space but not momentum*, w/same Profile Functions as  $f$ /Lattice Hamiltonian, or (b) fitting deformed local Fermi-D
- Coulomb Potential through Relaxation-Method Solution of Poisson Eq

Literature: NPA533(91)712, NPA673(00)375



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# Boltzmann Equation

Reaction simulated in terms of a set of **semi-phenomenological Boltzmann equations** for phase-space distributions  $f$  of  $N_s$ ,  $\pi_s$ ,  $\Delta_s$ ,  $N^*_s$ ,  $ds \dots$ :

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = I$$

where the single-particle energies  $\epsilon$  are given in terms of the net **energy functional**  $E\{f\}$  by,

$$\epsilon(\mathbf{p}) = \frac{\delta E}{\delta f(\mathbf{p})}$$

In the local cm, the mean potential is  $U_{opt} = \epsilon - \epsilon_{kin}$  and  $\epsilon_{kin} = \sqrt{p^2 + m^2}$ .





## Energy Functional

The functional:

$$E = E_{vol} + E_{gr} + E_{iso} + E_{Coul}$$

where

$$E_{gr} = \frac{a_{gr}}{\rho_0} \int d\mathbf{r} (\nabla \rho)^2$$

For covariant volume term, ptcle velocities parameterized in local frame:

$$v^*(p, \rho) = \frac{p}{\sqrt{p^2 + m^2} \left( 1 + c \frac{\rho}{\rho_0} \frac{1}{(1 + \lambda p^2/m^2)^2} \right)^2}$$

precluding a supraluminal behavior (PD *et al* PRL81(98)2438), with  $\rho$  - baryon density. The 1-ptcle energies are then

$$\epsilon(p, \rho) = m + \int_0^p dp' v^* + \Delta\epsilon(\rho)$$

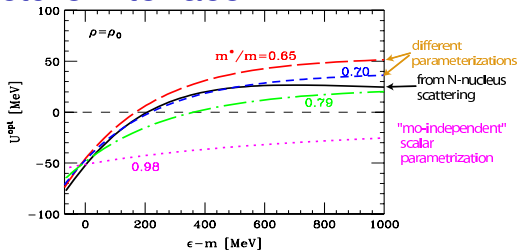
Parameters in the velocity varied to yield different optical potentials characterized by values of effective mass,

$$m^* = p_F / v_F.$$



## Structure Interface

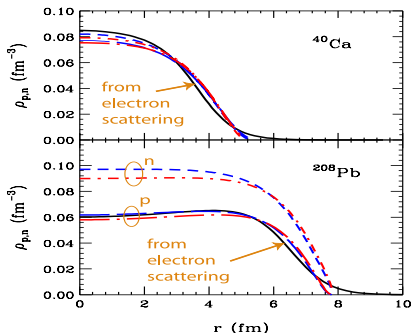
Potential from  
p-scattering  
(Hama *et al*  
PRC41(90)2737)  
& parameterizations



Ground-state densities from  
electron scattering and from  
functional minimization.  
From  $E(f) = \min$  :

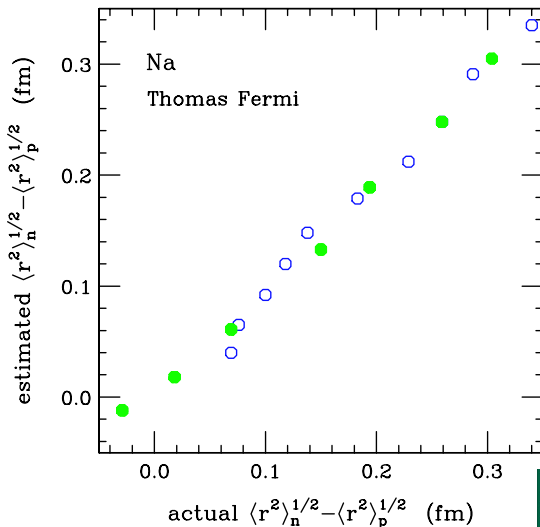
$$0 = \epsilon \left( \rho^F(\rho) \right) - 2 a_{gr} \nabla^2 \left( \frac{\rho}{\rho_0} \right) - \mu$$

⇒ Thomas-Fermi eq.



# Finer Details of Thomas-Fermi Solutions

Neutron skin:  
macroscopic  
theory vs  
Thomas-Fermi  
w/sym energy  
variation



## Practical Aspects of Dynamics

Pseudoparticle representation for the phase-space distribution

$$f(\mathbf{r}, \mathbf{p}, t) = \frac{1}{\mathcal{N}} \sum_{i=1}^{A \cdot \mathcal{N}} \delta(\mathbf{r} - \mathbf{r}_i(t)) \delta(\mathbf{p} - \mathbf{p}_i(t))$$

Space divided in cells of volume  $\Delta V$ . **Lattice hamiltonian** (Lenk&Pandharipande PRC39(89)2242) from energy densities at cell nodes  $\mu$

$$E = \Delta V \sum_{\nu} e_{\nu} \{f_{\mu}\}$$

where  $e$  is energy density and

$$f_{\nu} = \frac{1}{\mathcal{N}} \sum_i S(\mathbf{r}_{\nu} - \mathbf{r}_i) \delta(\mathbf{p} - \mathbf{p}_i(t))$$

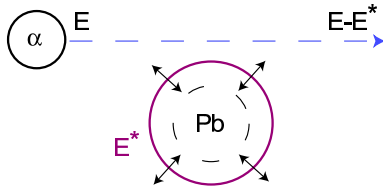
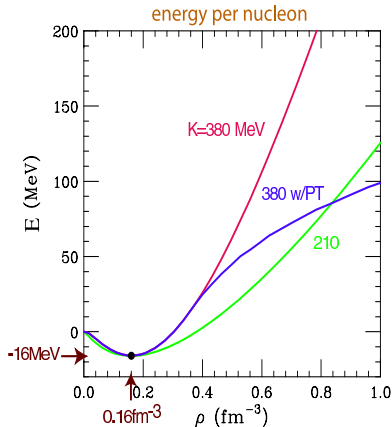
$S$  localized profile function and

$$\dot{\mathbf{r}}_i(t) = \frac{\partial E}{\partial \mathbf{p}_i} \quad \dot{\mathbf{p}}_i(t) = -\frac{\partial E}{\partial \mathbf{r}_i}$$

integrate the l.h.s. of the Boltzmann eq. (Vlasov).



# Incompressibility from Vibrations?



$$E^* = \hbar\Omega = \hbar \sqrt{\frac{K}{m_N \langle r^2 \rangle_A}}$$

Problem: surface, Coulomb, isospin imbalance

⇒ all that in Boltzmann eq.

$$K = 9 \rho_0^2 \frac{d^2}{d\rho^2} \left( \frac{E}{A} \right) = R^2 \frac{d^2}{dR^2} \left( \frac{E}{A} \right)$$

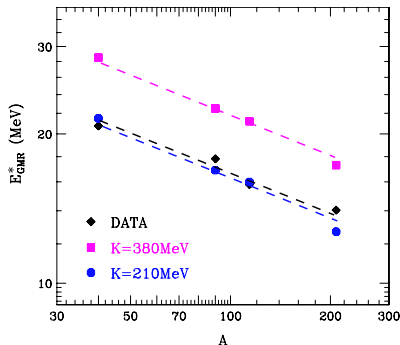
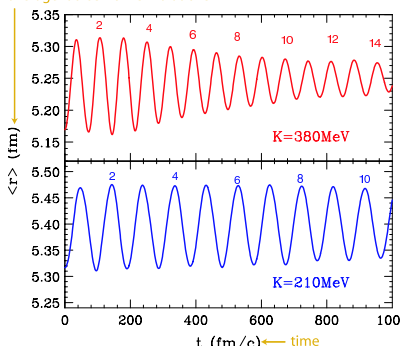


# Monopole Oscillations

## Pb Oscillations

$$E_{GMR}^* = \hbar\Omega$$

average radius from simulations



data Youngblood, Garg *et al.*

$$\Rightarrow K = (225 - 240) \text{ MeV}$$



## Collision Rates

Collision rate incorporates effects of interactions of different particle numbers:

$$I = I_2 + I_3 + \dots$$

2-body collision rate

$$I_2 = \int |\mathcal{M}_{12 \rightarrow \dots}|^2 \delta(\mathbf{P}' - \mathbf{P}) \delta(E' - E) f_1 f_2 (1 - f'_1) \dots$$

3-body collision rate

$$I_3 = \int |\mathcal{M}_{123 \rightarrow \dots}|^2 \delta(\mathbf{P}' - \mathbf{P}) \delta(E' - E) f_1 f_2 f_3 \dots$$

3 nucleons required to form a deuteron, 4 nucleons to form a triton ...



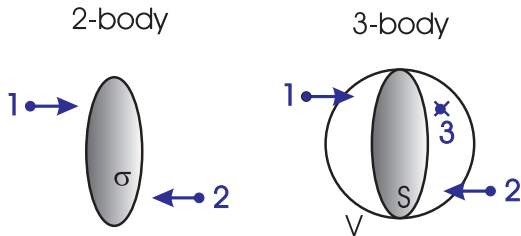
## 3-Body Collisions

Net 2-body collision rate:

$$\int dP_f |\mathcal{M}_{12 \rightarrow \dots}|^2 \delta(\mathbf{P}' - \mathbf{P}) \delta(E' - E) = \sigma_{12} v_{12}$$

Net 3-body collision rate:

$$\int dP_f |\mathcal{M}_{123 \rightarrow \dots}|^2 \delta(\mathbf{P}' - \mathbf{P}) \delta(E' - E) = \mathcal{V} S v = \mathcal{V}_3 \sigma_{12} v_{12}$$

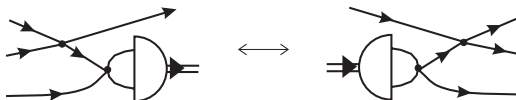




# Deuteron Production

Detailed balance:

$$|\overline{\mathcal{M}}^{npN \rightarrow Nd}|^2 = |\overline{\mathcal{M}}^{Nd \rightarrow Nnp}|^2 \propto d\sigma^{Nd \rightarrow Nnp}$$



Thus, production can be described in terms of breakup.

$$d\sigma^{Nd \rightarrow Nnp} \propto \sigma_{np} |\phi_d(p)|^2 \propto \sigma_{np} \mathcal{V}_N$$

Modified impulse approximation employed.

(PD&Bertsch NPA533(91)712)

Tritons and helions produced in a similar manner in 4-nucleon collisions.



# Low-Energy Comparison to INDRA

$^{129}\text{Xe} + ^{119}\text{Sn}$  at  
50 MeV/nucleon

points - data

Gorio

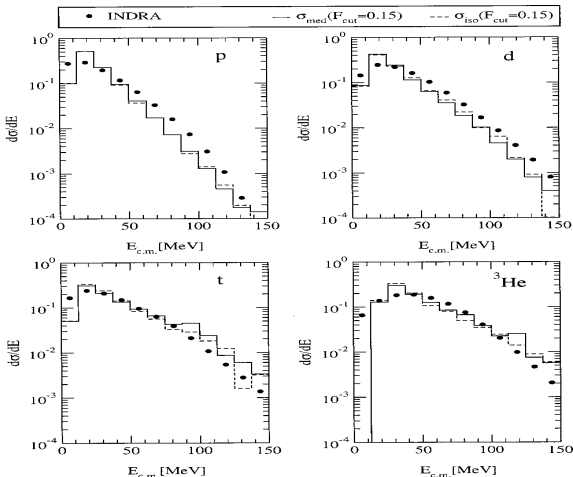
EPJA7(00)245

histograms -

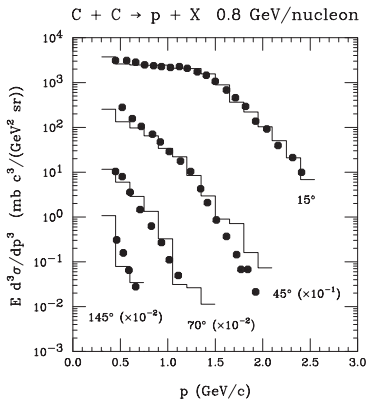
calculations

Kuhrts

PRC63(01)034605

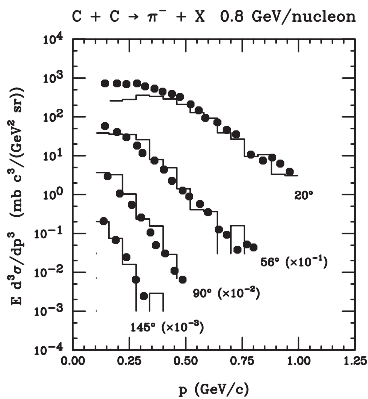


# High-Energy Inclusive Data



proton

&amp;



pion spectra

points - data Nagamiya PRC24(81)971  
histograms - calculations

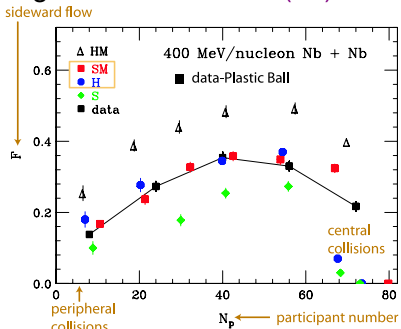


# Potential Ambiguity in Conclusions

When observables are sensitive to bulk properties, they are usually sensitive to few properties at once.

⇒ For progress, one needs to look for dedicated observables sensitive to one particular observable.

E.g.: Pan&PD PRL70(93)2063



SM - strong dependence of  $\epsilon$  on  $p$

H - strong dependence of  $\epsilon$  on  $\rho$



## Stopping: $\sigma_{NN}$ & Viscosity

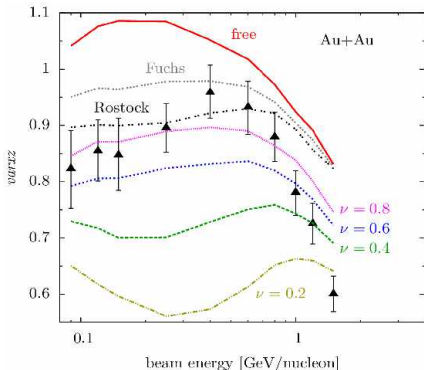
- Central symmetric collisions from 0.09 to 1.5 GeV/u
- Stopping observables such as

$$varxz = \frac{\Delta y_x}{\Delta y_z}$$

- Free CS overestimates stopping
- Different CS modifications tried
- Tempered CS works best

$$\sigma \lesssim \nu \rho^{-2/3}$$

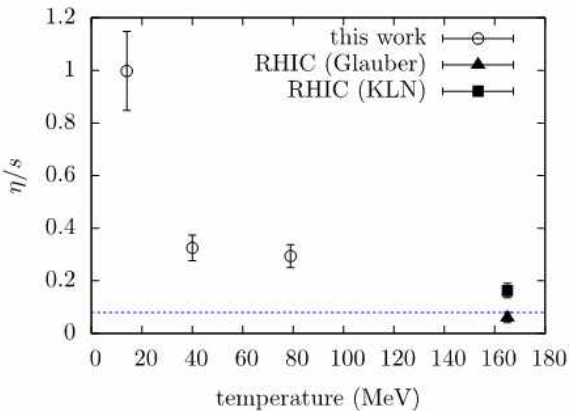
with  $\nu \sim 0.7$



Reisdorf *et al* [FOPI]  
PRL92(04)232301  
NPA848(10)366



# Viscosity-to-Entropy Ratio



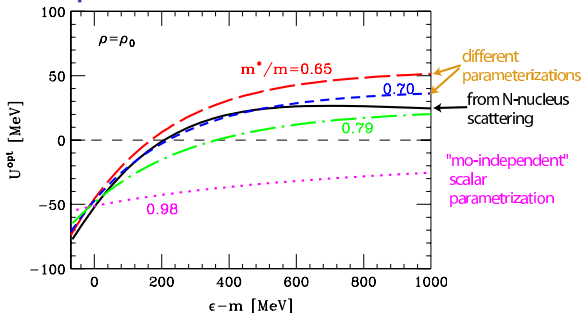
Viscosity from reduced in-medium cross-sections  
RHIC: [Bernhard \*et al\* PRC91\(15\)054910](#)



# Momentum Dependence of Mean Field

Nucleon-nucleus scattering gives access to the mean field at densities  $\rho \lesssim \rho_0$

Hama *et al*  
PRC41(90)2737



Evidence for momentum dependence in reactions?

Access to momentum dependence at  $\rho > \rho_0$ ?

$$U^{opt} = \epsilon - \epsilon^{kin} \quad \mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{p}}$$

$$\mathbf{v} = \frac{\partial \epsilon^{kin}}{\partial \mathbf{p}} + \frac{\partial U^{opt}}{\partial \mathbf{p}} = \mathbf{v}^{kin} + \frac{\partial U^{opt}}{\partial \mathbf{p}} > \mathbf{v}^{kin}$$

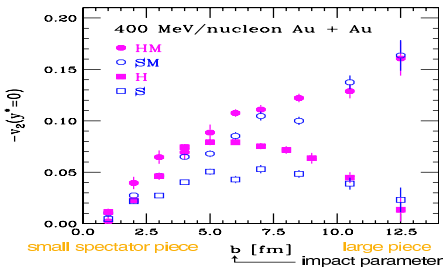
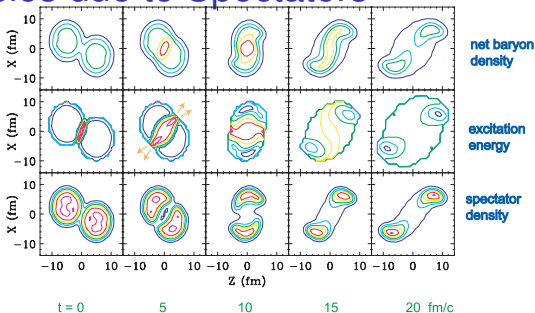
How to assess the in-medium velocities in central reactions??



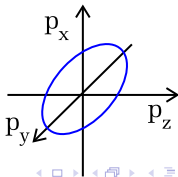
# Anisotropies due to Spectators

Spectator nucleons:  
weakly affected by  
reaction, proceed at  
unaltered velocity

Participants: matter  
undergoes violent  
process, compression,  
excitation & expansion



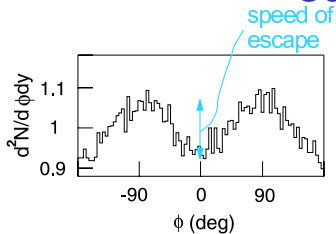
$$v_2 = \langle \cos(2\phi) \rangle \quad \phi - \text{relative to reaction plane}$$





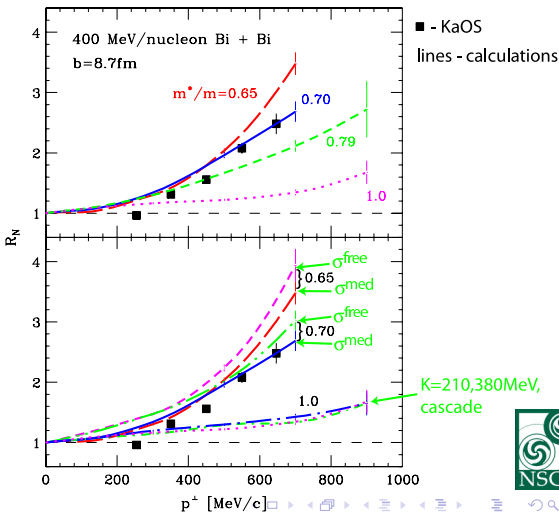
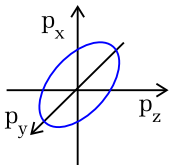
# Comparison to Data

$$R_N = \frac{N(-90^\circ) + N(90^\circ)}{N(0^\circ) + N(180^\circ)}$$



data: KaOS Brill *et al*  
ZPA355(96)61

More ptcles escape in direction perpendicular to the reaction plane



# Supranormal Densities?

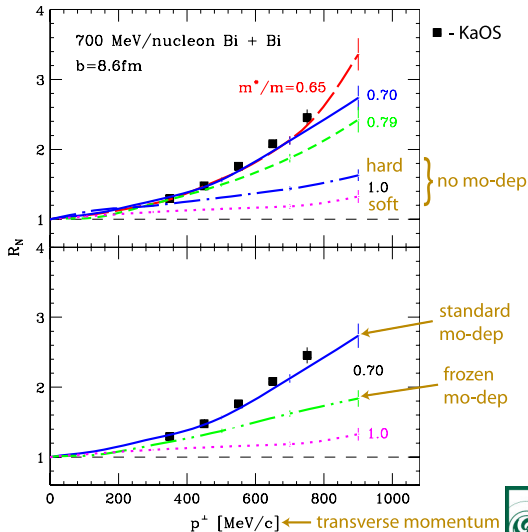
Other beam energies??

Are we just testing the momentum dependence in the vicinity of  $\rho_0$ ??

Test: Max.  $\rho$  in midperipheral collisions at 400, 700 and 1000 MeV/nucleon:  $\rho/\rho_0 \approx 1.85$ , 2.20 and 2.40, respectively. But do they matter??

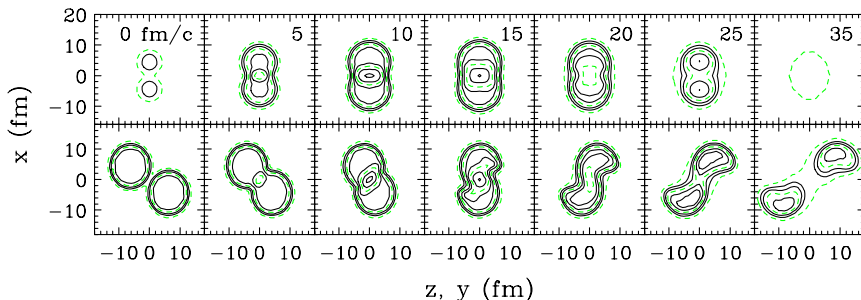
⇒ Let us make the momentum dependence at  $\rho > \rho_0$  follow dependence at  $\rho_0$ . MF where velocity

ceases to change above  $\rho_0$ :  $v^*(p, \rho) = v^*(p, \rho_0)$  for  $\rho > \rho_0$ .



# Why Sensitivity to $\rho > \rho_0$ in Transverse Directions??

Contour plots of the density in the reaction plane (bottom) and in the plane  $\perp$  to the beam (top) for Bi+Bi at 400MeV/u:



Fast ptcls emitted transversally, around  $t \sim 15$  fm/c, directly from high- $\rho$  matter!

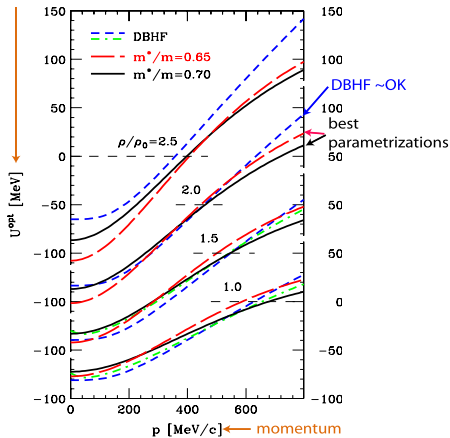
PD NPA673(00)375



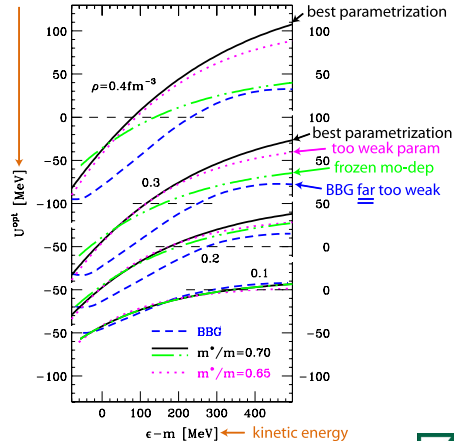
# Comparison to Microscopic Calculations

Optical-potential  $U = \epsilon - \epsilon_{kin}$  compared to microscopic

nucleon optical potential



nucleon optical potential



Dirac-Brueckner-Hartree-Fock  
 Machleit *et al.* PRC48(93)2707

Bethe-Brueckner-Goldstone  
 Lombardo *et al.* PLB334(94)12



## Central Reactions

**Reaction plane:** plane in which the centers of initial nuclei lie.

**Spectators:** nucleons in the reaction periphery, little disturbed by the reaction.

**Participants:** nucleons that dive into compressed excited matter.

Nuclear EOS deduced from the features of collective flow in reactions of heavy nuclei.

Collective flow: motion characterized by significant space-momentum correlations, deduced from momentum distributions of particles emitted in the reactions.

Euler eq. in  $\vec{v} = 0$  frame:

$$m_N \rho \frac{\partial}{\partial t} \vec{v} = -\vec{\nabla} p$$



# EOS and Flow Anisotropies

EOS assessed through reaction plane anisotropies characterizing particle collective motion.

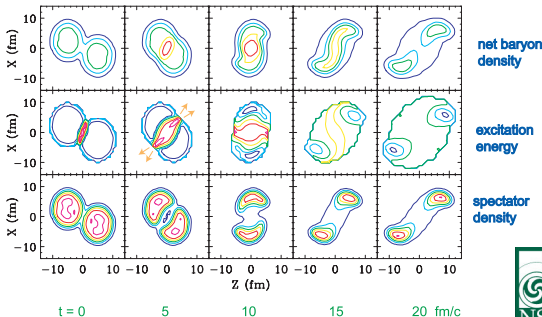
Hydro? Euler eq. in  $\vec{v} = 0$  frame:  $m_N \rho \frac{\partial}{\partial t} \vec{v} = -\vec{\nabla} p$

where  $p$  - pressure. From features of  $v$ , knowing  $\Delta t$ , we may learn about  $p$  in relation to  $\rho$ .  $\Delta t$  fixed by spectator motion.

For high  $p$ , expansion rapid and much affected by spectators.

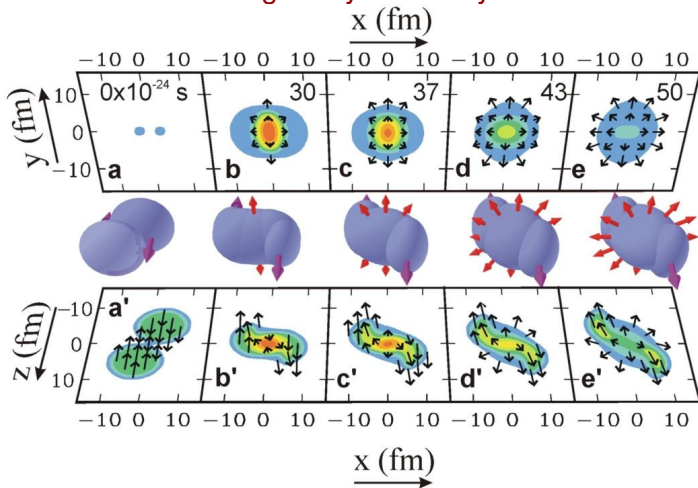
For low  $p$ , expansion sluggish and completes after spectators gone.

Simulation by L. Shi



# Medium-Energy Collisions of Heavy Nuclei

Thermalized matter at high baryon density! 2 GeV/u Au+Au



Top panels: pressure  $\perp$  to beam axis (up to 90 MeV/fm<sup>3</sup>) + flow

Bottom panels: density (up to  $3\rho_0$ ) in reaction plane + flow

# Sideward Flow Systematics

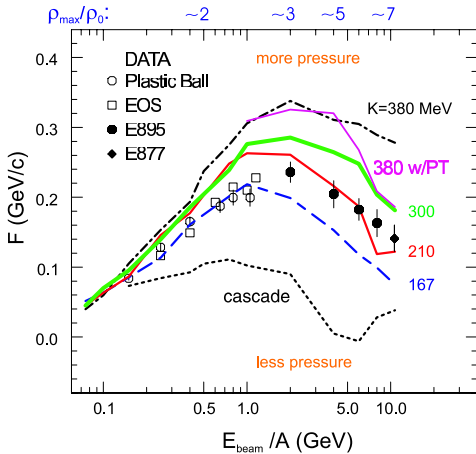
Deflection of forwards and backwards moving particles away from the beam axis, within the reaction plane.

Au + Au Flow  
Excitation Function

Note:  $K$  used as a label

PD, Lacey & Lynch

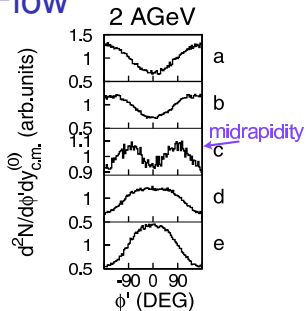
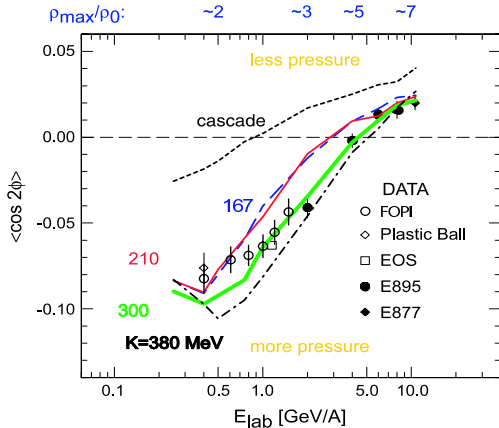
The sideward-flow observable results from dynamics that spans a  $\rho$ -range varying with the incident energy.





## 2<sup>nd</sup>-Order or Elliptic Flow

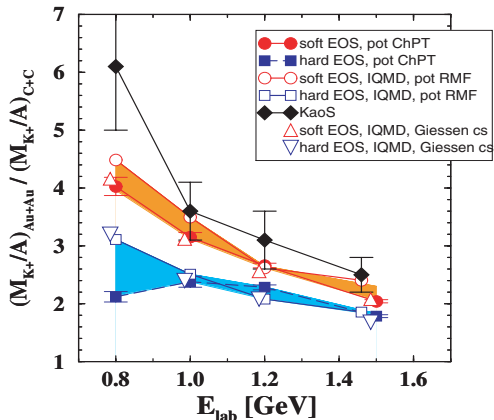
Another anisotropy, studied at midrapidity:  
 $v_2 = \langle \cos 2\phi \rangle$ , where  $\phi$  is azimuthal angle  
 relative to reaction plane.



Au+Au  $v_2$   
 Excitation Function



# Subthreshold Meson ( $K/\pi$ ) Production



Ratio of kaons per participant nucleon in Au+Au collisions to kaons in C+C collisions vs beam energy

filled diamonds: KaoS data

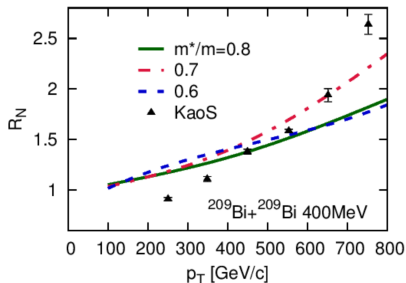
open symbols: theory  
*Fuchs et al*

Kaon yield sensitive to EOS because multiple interactions needed for production, testing density.

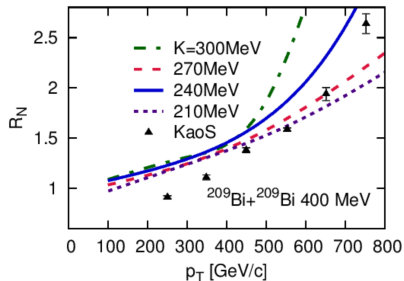
The data suggest a relatively soft EOS.



# Sensitivity of Elliptic Flow to $m^*/m$ and $K$



$K = 270$  MeV  
and changing  $m^*/m$

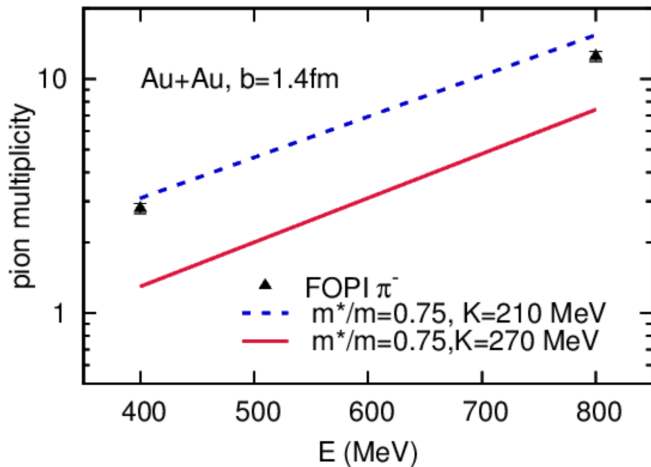


$m^*/m = 0.7$   
and changing  $K$

Hysteresis in both cases due to competition between density and momentum dependence



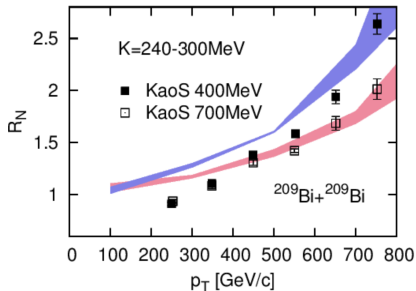
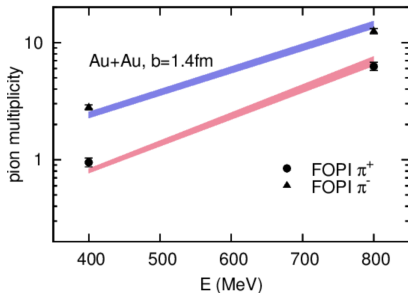
# Sensitivity of $M_\pi$ to Incompressibility $K$



$m^*/m = 0.75$  and changing  $K$



# Raising $K$ Allows to Describe Both $M_\pi$ and $v_2!$



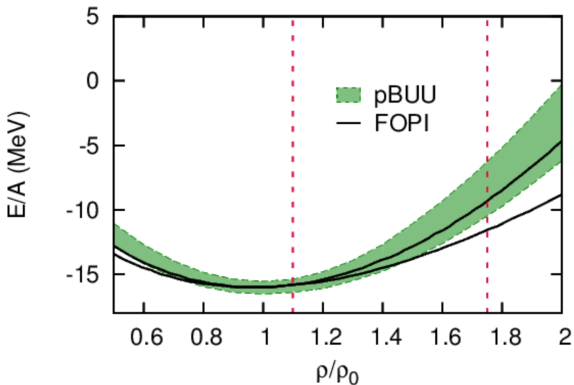
Bands for  $K = (240 - 300)$  MeV & optimal  $m^*/m$

→ Constraints on EOS, at moderately supranormal densities,  
à la LeFèvre *et al*



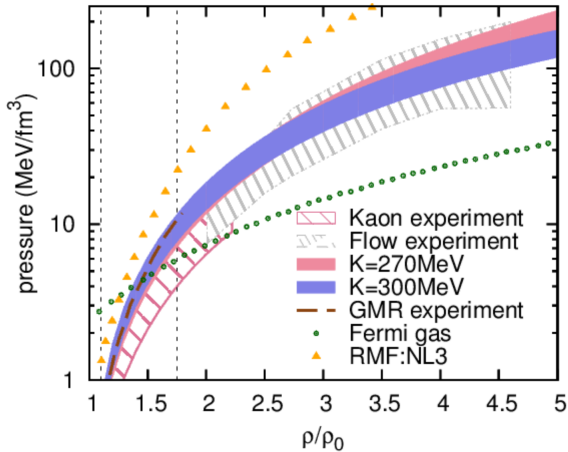
# Energy Per Nucleon

## Symmetric Matter



# Pressure

## Symmetric Matter



## Principal Features Again

- Solution of Boltzmann Eq
- 1-Particle Energies from Energy Functional
  - Volume (incl Momentum), Gradient, Isospin, Coulomb Terms
- Covariance
  - Covariant: Volume (incl Momentum) Term in Energy, Collisions
  - Noncovariant: Gradient, Isospin, Coulomb Terms in Energy
  - Employed (so far) up to 20 GeV/nucleon
- Pions contribute to Symmetry Energy
- Spectral functions of  $\Delta$  and  $N^*$  Resonances in adiabatic approximation
  - Detailed Balance for Broad Resonances
- $A = 2, 3$  Clusters produced in Multinucleon Collisions
  - Cluster Break-Up Data used in describing Production

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